Theory of Numbers

1. The stronger Goldbach conjecture states that any even integer greater than 7 can be written as the sum of two different prime numbers [1]. For such representation of the even numbers 126, the largest possible difference between the two primes is

   (A) 112 **(B)** 100 **(C)** 92 **(D)** 88 **(E)** 80

2. Define $n_a!$ for n and a positive to be

   $$n_a! = n(n-a)(n-2a)(n-3a)(n-4a)...(n-ka)$$

   where k is the greatest integer for which $n > ka$. Then the quotient $\frac{72_8!}{18_2!}$ is equal to

   (A) 4^5 **(B)** 4^6 **(C)** 4^8 **(D)** 4^9 **(E)** 4^12

3. The number of sets of two or more consecutive positive integers whose sum is 100 is

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

   (A) 0 **(B)** 1 **(C)** 49 **(D)** 50 **(E)** 51

5. The integers greater than one are arranged in five columns as follows:

	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
	...	...	...	...

   (Four consecutive integers appear in each row; in the first, third and other odd numbered rows, the integers appear in the last four columns and increase from left to right; in the second, fourth and other even numbered rows, the integers appear in the first four columns and increase from right to left.)

   In which columns will the number 1000 fall?

   (A) first **(B)** second **(C)** third **(D)** fourth **(E)** fifth

[1]The regular Goldbach conjecture states that any even integer greater than 3 is expressible as a sum of two primes. Neither this conjecture nor the stronger version has been settled.

6. What is the smallest prime numbers dividing the sum $3^{11} + 5^{13}$?

   (A) 2 **(B)** 3 **(C)** 5 **(D)** $3^{11} + 5^{13}$ **(E)** none of these

7. How many primes less than 100 have 7 as the ones digit? (Assume the usual base 10 representation.)

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. Three primes, p, q and r satisfy $p + q = r$ and $1 < p < q$. Then p equals

   (A) 2 **(B)** 3 **(C)** 7 **(D)** 13 **(E)** 17

9. If $p \geq 5$ is a prime number, then 24 divides $p^2 - 1$ without remainder

   (A) never **(B)** sometimes only **(C)** always **(D)** only if $p = 5$ **(E)** none of these

Bibliography

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2. Berzsenyi, George, and Stephen B. Maurer. The Contest Problem Book V. American High School Mathematics Examinations and American Invitational Mathematics Examinations: 1983-1988. Problems and solutions compilad and augmented by George Berzsenyi of Rose-Hulman Institute of Technology and Stephen B. Maurer of Swartmore College.