

8. If

$$N = \frac{\sqrt{\sqrt{5} + 2} + \sqrt{\sqrt{5} - 2}}{\sqrt{\sqrt{5} + 1}} - \sqrt{3 - 2\sqrt{2}}$$

then N equals

- (A) 1 (B) $2\sqrt{2} - 1$ (C) $\frac{\sqrt{5}}{2}$ (D) $\sqrt{\frac{5}{2}}$ (E) none of these

9. If $a = 1$, $b = 10$, $c = 100$ and $d = 1000$, then

$$(a + b + c - d) + (a + b - c + d) + (a - b + c + d) + (-a + b + c + d)$$

is equal to

- (A) 1111 (B) 2222 (C) 3333 (D) 1212 (E) 4242

10. Let

$$\frac{3}{2} + \frac{5}{4} + \frac{9}{8} + \frac{17}{16} + \frac{33}{32} + \frac{65}{64} - 7$$

is equals

- (A) $-\frac{1}{64}$ (B) $-\frac{1}{16}$ (C) 0 (D) $\frac{1}{16}$ (E) $\frac{1}{64}$

11. The product of $\sqrt[3]{4}$ and $\sqrt[4]{8}$ equals

- (A) $\sqrt[7]{12}$ (B) $2\sqrt[7]{12}$ (C) $\sqrt[7]{32}$ (D) $\sqrt[12]{32}$ (E) $2\sqrt[12]{32}$

12. To the nearest thousandth, $\log 2 = 0,301$ and $\log 3 = 0,477$. Which of the following is the best approximation of $\log_5 10$?

- (A) $\frac{8}{7}$ (B) $\frac{9}{7}$ (C) $\frac{10}{7}$ (D) $\frac{11}{7}$ (E) $\frac{12}{7}$

13. If $a = \frac{1}{2}$ and $(a+1)(b+1)=2$, then the radian measure of $\arctan a + \arctan b$ is equals

- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{5}$ (E) $\frac{\pi}{6}$

14. The sum $\sqrt[3]{5 + 2\sqrt{13}} + \sqrt[3]{5 - 2\sqrt{13}}$, equals

- (A) $\frac{3}{2}$ (B) $\frac{\sqrt[3]{65}}{4}$ (C) $\frac{1 + \sqrt[6]{13}}{2}$ (D) $\sqrt[3]{2}$ (E) none of these

15. Evaluate $(x^x)^{(x^x)}$, at $x=2$

- (A) 16 (B) 64 (C) 256 (D) 1024 (E) 65 536

16. Find the units digits in the decimal expansion of

$$(15 + \sqrt{220})^{19} + (15 + \sqrt{220})^{82}$$

- (A) 0 (B) 2 (C) 5 (D) 9 (E) none of these

17. If

$$\log_7 [\log_3 (\log_2 x)] = 0$$

then $x^{-\frac{1}{2}}$ equals

- (A) $\frac{1}{3}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{3\sqrt{3}}$ (D) $\frac{1}{\sqrt{42}}$ (E) none of these

18. The value of

$$\frac{1000^2}{252^2 - 248^2}$$

is

- (A) 62 500 (B) 1000 (C) 500 (D) 250 (E) $\frac{1}{2}$

19. The product of all real roots of the equation $x^{\log x} = 10$ is

- (A) 1 (B) -1 (C) 10 (D) 10^{-1} (E) none of these

20. The value of

$$\frac{2\sqrt{6}}{\sqrt{2} + \sqrt{3} + \sqrt{5}}$$

is

- (A) $\sqrt{2} + \sqrt{3} - \sqrt{5}$ (C) $\sqrt{2} + \sqrt{3} + \sqrt{6} - 5$ (E) $\frac{1}{3}(\sqrt{3} + \sqrt{5} - \sqrt{2})$
 (B) $4 - \sqrt{2} - \sqrt{3}$ (D) $\frac{1}{2}(\sqrt{2} + \sqrt{5} - \sqrt{3})$

21. Let

$$\frac{\sin 10^\circ + \sin 20^\circ}{\cos 10^\circ + \cos 20^\circ}$$

equals

- (A) $\tan 10^\circ + \tan 20^\circ$ (C) $\frac{1}{2}(\tan 10^\circ + \tan 20^\circ)$ (E) $\frac{1}{4}\tan 60^\circ$
 (B) $\tan 30^\circ$ (D) $\tan 15^\circ$

22. Simplify $(\sqrt[6]{27} - \sqrt{6\frac{3}{4}})^2$
- (A) $\frac{3}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{3\sqrt{3}}{4}$ (D) $\frac{3}{2}$ (E) $\frac{3\sqrt{3}}{2}$
23. The population of the United States in 1980 was 226 504 825. The area of the country is 3 615 122 square miles. There are $(5280)^2$ square feet in one square mile. Which number below best approximates the average number of square feet per person?
- (A) 5000 (B) 10 000 (C) 50 000 (D) 100 000 (E) 500 000
24. The value of $\frac{2^1+2^0+2^{-1}}{2^{-2}+2^{-3}+2^{-4}}$ is
- (A) 6 (B) 8 (C) $\frac{31}{2}$ (D) 24 (E) 512
25. The value of $\sqrt{8} + \sqrt{18}$ equals
- (A) $\sqrt{26}$ (B) $2(\sqrt{2} + \sqrt{3})$ (C) 7 (D) $5\sqrt{2}$ (E) $2\sqrt{13}$
26. Evaluate the product
- $$(\sqrt{5} + \sqrt{6} + \sqrt{7})(\sqrt{5} + \sqrt{6} - \sqrt{7})(\sqrt{5} - \sqrt{6} + \sqrt{7})(-\sqrt{5} + \sqrt{6} + \sqrt{7})$$
- (A) 92 (B) 104 (C) 180 (D) 256 (E) 302
27. Compute
- $$\frac{(10^4 + 324)(22^4 + 324)(34^4 + 324)(46^4 + 324)(58^4 + 324)}{(4^4 + 324)(16^4 + 324)(28^4 + 324)(40^4 + 324)(52^4 + 324)}$$
- (A) 89 (B) 317 (C) 373 (D) 689 (E) 1205
28. Simplify $\sin(x - y) \cos y + \cos(x - y) \sin y$.
- (A) 1 (B) $\sin x$ (C) $\cos x$ (D) $\sin x \cos 2y$ (E) $\cos x \cos 2y$
29. If $A = 20^\circ$ and $B = 25^\circ$, then the value of $(1 + \tan A)(1 + \tan B)$ is
- (A) $\sqrt{3}$ (C) $1 + \sqrt{2}$ (E) none of these
 (B) 2 (D) $2(\tan A + \tan B)$
30. Which of the following is closest to $\sqrt{65} - \sqrt{63}$?
- (A) 0.12 (B) 0.13 (C) 0.14 (D) 0.15 (E) 0.16

31. Find all the numbers of four digits with the following property: if we pass the first number to the last place and we subtract the original number we obtain 7893.
- (A) 1986 (B) 1987 (C) 1988 (D) 1997 (E) 1999

Bibliography

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